Endogenized bargaining power

- A theoretical analysis of union-firm bargaining

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Abstract

This thesis investigates a union-firm Nash bargaining solution when the bargaining power is endogenized. Applying the Rubinstein bargaining model, the bargaining power depends on the membership and the after-tax interest rate. The bargaining power for the union increases in the membership and the interest rate, but decreases in taxes. The membership is solved for by letting the union members impose reputational costs on non-members, and the tax-rate is solved for by having the government adjust the tax-rate to keep the unemployment at a constant level. I find that a fall in reputational costs decreases wages, unemployment and welfare for the union members, but that the decrease in welfare and wages is lower when the government adjusts taxes to keep unemployment constant.

Contents

1 Introduction

Trade unions can be of two types: Open shop or closed shop. In a closed shop union, the firm hires union members before non-members. This discrimination isn't assumed in an open shop union, where membership must be determined by something other than a higher probability of employment. There have been made efforts to model union-firm bargaining with an open shop union (Booth & Chatterji, 1993): In the paper even though membership is endogenized, it still assumes a monopoly union model. This means that a trade union with 1% of the population as members have the same bargaining power as one with 100%. I will slack the monopoly union assumption in this thesis to allow the membership to relate to the bargaining power. This is possible because the monopoly union model is a special case of bargaining model where the trade union have all the bargaining power (Manning, 1987).

In the economic literature it is common to model the wage-outcome as a result of a bargaining process between a union and a firm. The firm wants to pay as low a wage as possible, while the union wants the opposite. The share of the gain from cooperating depends on each players bargaining power. The bargaining power of a firm and union dictates the outcome, so understanding what determines the bargaining power is of economic interest.

In this thesis I will construct an open shop union-firm bargaining model with endogenized bargaining power. I will use the Nash bargaining solution to arrive at a wage-outcome, and the Rubinstein bargaining model to derive an economic interpretation of the Nash bargaining solution. The bargaining setup is an efficient bargaining model, where the firm and union bargain over wages and employment.

The workers have an incentive to free-ride in an open shop union framework unless there is something that differentiates members from non-members. In (Booth & Chatterji, 1993) the labour force differs by a positive reputation benefit of joining the union, which determines the union membership. The membership will in this thesis be determined not by a positive reputational benefit, but by a negative reputational cost of not joining the union.

This thesis will examine the wage bargaining outcome, when bargaining power is endogenized. I will in section 2 give a presentation of the model. Section 3 will go into details with the Nash bargaining solution, and how the union members and the tax rate affect bargaining outcome. I will in section 4 introduce heterogeneity between the workers, and solve for the membership. Section 5 will examine the comparative statics of the model. I will in section 6 go into detail in the case of a falling membership, and examine what a government can do to remedy the effects. Section 7 concludes the thesis.

2 Model presentation

The purpose of this section is to present the objective functions for the union and firm, and solve for the labor demand in an efficient bargaining framework.

2.1 Union utility

It is assumed that all workers have risk-neutral indirect utility functions, $V(I) = I$. If the workers join the union, and are employed, they pay union dues equal to α. The unemployed union members don't pay union dues. The employment decision is a random draw from the population of workers. It is common in the economic literature to assume that the objectives for the union is to maximize the expected (indirect) utility function (Booth, 1995, p. 89), and this will also be assumed in this thesis. The union's utility function is then

$$U = L(w - \alpha) + (1 - L)b \tag{1}$$

2.2 Profit function of the firm

It is assumed that the production function only takes labor as input, and that the output elasticity is $\frac{1}{2}$. The firm pays a wage rate, w, to the employed workers. The price of the good is normalized to 1. The firm's profit function is then

$$\pi = \sqrt{L} - wL \tag{2}$$

The firm and unions simultaneously decide wages and employment in an efficient bargaining model. The marginal rates of substitution between employment and wages are equalized in a Pareto efficient outcome. This can be found by maximizing the utility of one player, while holding the other player's utility constant. The Lagrangian is

$$\mathcal{L} = \sqrt{L} - wL - \lambda \left(L \left(w - \alpha \right) + \left(1 - L \right) b - \overline{U} \right) \tag{3}$$

The first order conditions yields

$$\frac{\partial \mathcal{L}}{\partial L} = \frac{1}{2\sqrt{L}} - w - \lambda\left(w - \alpha - b\right) = 0 \tag{4}$$

$$\frac{\partial \mathcal{L}}{\partial w} = -L - \lambda L = 0 \Rightarrow \lambda = -1 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = L\left(w - \alpha\right) + \left(1 - L\right)b - \overline{U} = 0 \tag{6}$$

Inserting the expression for λ from (5) into (4) and isolating for L yields the labor demand.

$$L^{*} = \left(\frac{1}{2(\alpha + b)}\right)^{2} \tag{7}$$

where the range for $\alpha + b$ is

$$(\alpha + b) \in [1/2, \infty) \tag{8}$$

because the population of workers is normalized to 1.

It is assumed that the players operate in an open shop framework. This means that the firm doesn't discriminate workers based on membership of the union. The open-shop framework differs from the closed-shop framework, where the union forces the firm to employ union members first. The decision to be a union member can't therefore be about the higher probability of employment. L individuals will be randomly drawn out of the population for employment. The population is normalized to 1, which means that L is the probability for any worker to be employed, and $1 - L$ is the probability of unemployment.

2.3 Government involvement

The government is an outside player, that can influence the bargaining outcome in its favor. The wage outcome affects the utility of the voters through the wage and the unemployment, so a rational government wants to take actions, that maximizes the utility of its constituency. In the economics literature the behavior of democratic organizations with majority voting are often modeled so the electives try to maximize the utility of the median voter. It can be shown that this is the behavior that maximizes the probability of being (re-)elected (Black, 1948). The government controls the tax-rate on the interest rate, which is used to finance the unemployment benefits. It is assumed that the government runs a tight budget, so that $\tau r = u_b$, where τ is the tax-rate, r is the interest rate and u_b is the unemployment benefits.

3 Efficient bargaining

This section introduces the bargaining situation and -outcome. The first subsection introduces the Nash bargaining solution (NBS). The second subsection derive the NBS through the Rubinstein model, so that the bargaining power has an economic interpretation. The last subsection shows graphically the types of effects that changes the Nash bargaining solution.

3.1 The Nash bargaining solution

A bargaining situation is any situation where two players have an interest in cooperating, but have conflicting interest over how to share the gains from doing so (Muthoo, 1999, p. 1).

In the axiomatic approach to bargaining, the objective is to impose some assumptions (axioms) that defines the bargaining solution. John Nash showed that under the four axioms (invariance, Pareto optimality, independence of irrelevant alternatives and symmetry), the Nash bargaining solution is the solution to the maximization problem

$$\max_{v_i, v_j} \Omega = (v_i - \overline{v}_i)(v_j - \overline{v}_j) \tag{9}$$

where Ω is the Nash product, and v_i and $\overline{v}_i$ is the utility function and disagreement payoff for player i, respectively. The symmetry assumption will be weakened in this thesis, so the bargaining solution is the outcome that maximizes the generalized Nash product

$$\max_{v_i, v_j} \Omega = (v_i - \overline{v}_i)^{\beta_i} (v_j - \overline{v}_j)^{\beta_j} \tag{10}$$

The efficiency assumption leads to the joint utility of cooperating, J, being the sum of the utilities $J = v_i + v_j$. Using this as a constraint in the Lagrangian, with the logarithm of Ω yields

$$\mathcal{L} = \beta_i \ln(v_i - \bar{v}_i) + \beta_j \ln(v_j - \bar{v}_j) - \lambda(v_i + v_j - J) \tag{11}$$

The first order conditions of (11) is

$$\frac{\partial \mathcal{L}}{\partial v_i} = \beta_i \frac{1}{v_i - \overline{v}_i} - \lambda = 0 \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial v_j} = \beta_j \frac{1}{v_j - \overline{v}_j} - \lambda = 0 \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = v_i + v_j - J = 0 \tag{14}$$

By equalizing (12) and (13) to solve for v_i, and inserted into (14) yields

$$v_i = \bar{v}_i + \frac{\beta_i}{\beta_1 + \beta_2} \cdot (J - \bar{v}_1 - \bar{v}_2) \tag{15}$$

Inserting the expression for the union utility function and the profit into (10) gives the following Nash product.

$$(L\,(w - b - \alpha))^{\beta} \left(\sqrt{L} - wL\right)^{1-\beta} \tag{16}$$

where it is assumed that $\beta_1 + \beta_2 = 1$ and that the disagreement point is $d = (b, 0)$ for the union and firm, respectively. The joint utility is

$$J = U + \pi = \sqrt{L} - L\alpha + (1 - L)\,b \tag{17}$$

The optimal wage that maximizes the Nash product is

$$w = (b + \alpha)\,(1 + \beta) \tag{18}$$

(**See appendix A for calculations**)

The wage-outcome is a mark-up, $1+\beta$, on the alternative income $b+\alpha$. The alternative income consists of the inside option, b, and the union dues. The reason for this is that to forego the inside option, b, the members pay α so they need at least $b + \alpha$ to accept employment.

In the economics literature the parameter β is interpreted as bargaining power of the union (or firm). As Booth, Alison L, writes in (Booth, 1995, p. 124), this is an ad hoc interpretation. A way to endogenize β is through the Rubinstein model of bargaining.

3.2 Rubinstein

The following subsection follows the proof in (Muthoo, 1999, chp. 3)

The Rubinstein bargaining model is a method of solving alternating offers bargaining problems. The model was made by Ariel Rubinstein, and is used when there isn't any end period in a alternating offers bargaining problem. The model works as such:

One player proposes a share of the gains from cooperating. The other player then either accepts or rejects. If the other player rejects, it's that players turn to propose a share of the gains from cooperation. A key assumption is the discount factor, which is a player's utility loss from waiting. This can be interpreted as a depreciation of the gains from cooperating, or, as will be done in this thesis, a loss because of impatience.

In this thesis it will work as follows: The firm starts by proposing a share of some joint utility from cooperating, J. The union then either accepts or rejects the offer. If the union rejects the offer, then it will offer a share to the firm after Δ periods of time that the firm can either accept or reject. This back-and-forth continues until agreement is reached. It is assumed that the players discount future payoffs by some factor smaller than 1.

Let δ_π, and δ_U denote the discount factor of the firm and union, respectively. Let also V_π be the continuation value for the firm, and V_U the continuation value for the union. The continuation value is the non-discounted value that a player receives if an offer is rejected and bargaining continues. Two properties defines the equilibrium in the Rubinstein model:

Property 1: Stationarity: A player's offer is time-independent.

Property 2: No delay: The equilibrium offer of a player is accepted by the other player.

At any time period where it is the firm's time to give an offer, the firm will offer just enough so the union accepts. This is equal to the discounted continuation value, $\delta_U V_U$. The firms continuation value is then

$$V_\pi = J - \delta_U V_U$$

because by property 1 and 2, the equilibrium offer is offered in the first round. If, however, the union moves first, then the valuation offer is

$$V_U = J - \delta_\pi V_\pi$$

which follows from the symmetry of the game. Solving the two equations simultaneously yields the continuation value for player i as

$$V_i = J\frac{1-\delta_j}{1-\delta_U\delta_\pi} \tag{19}$$

which is the share of the joint utility player i receives. This follows from the stationarity and no delay properties. If the share wasn't the continuation value, then either the stationarity property or the no delay property is violated.

The continuous discount factor for player i is defined is $\delta_i \equiv e^{-(r_i\Delta)}$ where r_i is the cost of time for player i, and Δ is the time period between offers. Taking the taylor series around $\Delta = 0$ of the discount factor yields

$$e^{-(r_i\Delta)} \approx e^{-(r_i\cdot 0)} - r_i e^{-(r_i\cdot 0)}(\Delta - 0) = 1 - r_i\Delta$$

Which means that when the time period between consecutive offers approaches, but never reaches, zero the discount factor is

$$\delta_i = 1 - r_i\Delta$$

The continuation value for player i is in the case of a low Δ

$$\begin{aligned} V_i &= J\frac{1-\delta_i}{1-\delta_U\delta_\pi} \\ &= J\frac{1-e^{-(r_i\Delta)}}{1-e^{-(r_\pi+r_U)\Delta}} \\ &= J\frac{r_i\Delta}{(r_U+r_\pi)\Delta} \\ &= J\frac{r_i}{r_U+r_\pi} \end{aligned} \tag{20}$$

The share of the joint utility is determined by the relative eagerness of reaching an agreement. Player i receives a greater share, when player j gets more impatient.

3.2.1 Inside options

In bargaining theory payoffs by disagreeing are sorted into inside- and outside options. Outside options are the possible payoff whenever bargaining breaks down. This can for the labor force mean working for a different firm or being unemployed, and for the firm mean converting the production to a different non-conflicting sector. Inside options are the payoff received when the union and firm haven't come to an agreement, but bargaining hasn't broken down. In this thesis the players have insignificant outside options. This means that the outside options can be ignored.

Whenever the firm and union disagrees the members will, in the short term, strike. To keep the model simple, it is assumed that the strikers 'picket' outside to block entries of non-members to the workplace.

When the workers strike, they receive a strike pay equal to the unemployment benefit. The inside option for the members is therefore the unemployment benefit. At any time, t, the unemployment benefit is u_b. The production is stopped because of the 'picketing', so the firm's inside option is zero.

If the firm and union reach agreement at time t, then the discounted payoff to the union are

$$V_U \cdot e^{(-r_U \cdot \Delta t)} + \int_0^{\Delta t} u_b \cdot e^{(-r_U s)} ds = V_U e^{(-r_U \cdot \Delta t)} + \frac{u_b}{r_U}\left(1 - e^{(-r_U \Delta t)}\right) \tag{21}$$

The last term is the inside option for the union at time t. Taking the limit for $\Delta \to \infty$ yields the payoff as $\frac{u_b}{r_U}$ which have the following interpretation: If the union disagrees perpetually, then it receives a payoff of u_b/r_U. It then follows that the union will reject any offer below $\frac{u_b}{r_U}$. So in any subgame perfect equilibrium the firm must offer the union at least $\frac{u_b}{r_U}$ from the joint utility, J, before bargaining can continue. The surplus are $J - \frac{u_b}{r_U}$, which will be allocated according to (20). The utility share of the union is therefore

$$V_U^* = \frac{u_b}{r_U} + \frac{r_\pi}{r_\pi + r_U}\left(J - \frac{u_b}{r_U}\right) \tag{22}$$

which is the Nash bargaining solution with $b = \frac{u_b}{r_U}$ and $\beta = \frac{r_\pi}{r_\pi + r_U}$. This means that β is the relative eagerness to agree. When the firm gets more eager, meaning r_π increases, the union's proportion of the bargained share increases.

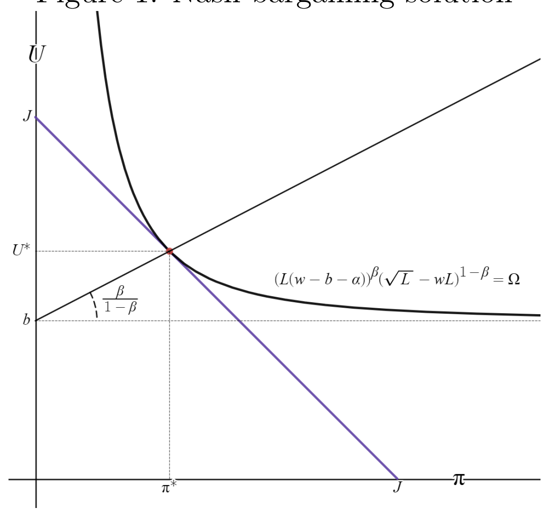
The cost of time, r_i, is the cost for player i of haggling. It is assumed that $r_U = 1 - \alpha M$, where M is the membership. αM is the amount of dues the union collects, and since haggling is costly, it is realistic that a union with more dues can pay to haggle a longer time, and so the cost of time falls with more union dues.

The firm's cost of time is its opportunity cost from receiving a payoff at a later time. If agreement is struck at a later time, then the cost is the gain the firm could have gotten from investing the payoff at the interest rate r. The interest rate is taxed by a tax rate, τ, so the cost of time is therefore $r_\pi = r(1 - \tau)$.

3.3 Graphical analysis of the Nash bargaining solution

From (22) it is easy to see, that if r_U and r_π both decreases such that β is constant, then the union will still get a greater share from the higher inside option. The inside option, b, will increase because of the lower cost of time for the union. Bargaining power therefore depends on both the relative- and the absolute eagerness to agree. To differentiate between the two effects I define the bargaining power to be β, and the effect from the absolute eagerness to agree to be the inside option.

Figure 1: Nash bargaining solution



In a bargaining situation there is a trade-off of utility between the players. This is what makes the game a bargaining situation. Under the assumption of efficient bargaining, there is a one-to-one trade-off between the utility of the union vs the firm. The Pareto frontier is therefore a straight line with a negative slope of 1. In figure 1 the Nash bargaining solution is drawn with the Nash product and a line through the inside option. The line through b and (π^*, U^*) has a slope of $\frac{U^*-b}{\pi^*} = \frac{\beta(J-b)}{(1-\beta)(J-b)} = \frac{\beta}{1-\beta}$.

There is three effects that changes the bargaining outcome. The bargaining power, β, can change, which represents a rotation of the line through b. The inside option, b, can change which graphically shifts the line through b, and the joint utility, J, can change, which represents a shift in the Pareto frontier. The exogenous parameters effect on the bargaining solution will be examined through these three effects in the section on comparative statics. Having $\tau r = u_b$ creates a trade-off between the inside option and the bargaining power of the union. A higher tax rate decreases the cost of time for the firm, which decreases the bargaining power for the union. The higher tax, however, increases the unemployment benefits, which increases the inside option. Graphically an upward shift of the line through b in figure 1 comes with the cost of a right rotation of the line.

4 Membership decision

This section introduces heterogeneity between the workers. The heterogeneity is used to solve for the workers decision to join the union. The first subsection introduces the assumptions needed to solve for the membership. The second subsection solves for the membership. The second subsection is inspired by (Booth & Chatterji, 1993), in which the membership decision depends on a reputational benefit of joining the union in a monopoly union model, and where the union executives uses the median voter model to maximize the probability of reelection.

4.1 Membership as public good

The benefits of union bargaining is non-excludable, because all workers earn the same wage independent of membership status. This is because of the open-shop framework assumption. The benefits of union bargaining is also non-rivalrous, because a higher wage for the i'th worker doesn't mean that other workers can't earn the same wage. Union bargaining is therefore a public good for the workers.

The utility share for the union increases in membership, so non-members are free-riders of the benefits of unionisation. The population of workers is continuously distributed from 0 to 1, so a single union member is infinitesimal compared to the population of members. It is therefore assumed that the membership decision is independent of the marginal increase in bargaining power with respect to members.

The assumption of independence between the membership decision and the marginal utility is a strong assumption, but it simplifies calculations significantly. A weakening of the assumption could be solved by finding the membership. This would be where the marginal expected utility with respect to membership is equal to marginal cost of membership, but that is outside the scope of this thesis.

4.2 Bad reputation for free-riders

The extension that follows is analogously to a cartel that sustains itself by imposing costs on deviators. The members enforces utility costs on non-members in the form of a bad reputation, θ. They do this to incentivize workers to join the union. The workers differ in the utility cost of a bad reputation. An individual who doesn't care much about her reputation, will experience lower costs from free-riding, compared to an individual who cares a lot about the approval from her colleagues. It is furthermore assumed that only employed workers can suffer reputational costs, because only they have colleagues.

It is assumed that the reputational cost, θ, is uniformly distributed between 0 and $1-\mu$. An individual worker only joins the union if her expected utility is higher from joining than from free-riding.

$$EU_{\text{Member}} \equiv L\left(w-\alpha\right)+\left(1-L\right)b \geq L(w-\theta_i)+\left(1-L\right)b \equiv EU_{\text{Non-Member}} \tag{23}$$

If follows directly from (23) that the i'th worker only joins the union if $\alpha \leq \theta_i$, meaning the cost of free-riding must be at least as high as the cost of joining. The marginal member is the one that is indifferent between joining or not, $\alpha = \theta_M$. The cumulative distribution function for a uniform distribution, $X \sim U(a,b)$, is

$$F(x) = \int_a^x \frac{1}{b-a} dw = \frac{x-a}{b-a} \tag{24}$$

Because $\theta \sim U\left(0, 1-\mu\right)$ the membership is

$$M = 1 - F(\theta_M) = 1 - \frac{\alpha}{1-\mu} \tag{25}$$

Taking the differential of M yields $dM = -\frac{d\alpha}{1-\mu} - \frac{\alpha}{(1-\mu)^2}d\mu$. The membership decreases in α and μ. An increase in μ represents a fall in reputational costs.

The cost of time for the union depends on the union dues. Taking the differential of r_U yields $dr_U = -\alpha \cdot dM - M \cdot d\alpha$. Inserting the expression for dM gives

$$dr_U = d\alpha\left(\frac{2\alpha}{1-\mu} - 1\right) + \frac{\alpha^2}{\left(1-\mu\right)^2}d\mu \tag{26}$$

which means that for $\alpha < \frac{1}{2}$ the marginal increase in total union dues are greater than the loss in total union dues from lower membership. For $\alpha > \frac{1}{2}$ the marginal increase in total union dues from a higher union due are smaller than the marginal loss in total union dues from fewer workers. The cost of time strictly increases in a lower reputational cost.

5 Comparative Statics

In this section there will be shown comparative statics. The effect on welfare will be computed for the union members only, because they are the population of interest. The first subsection provides some preliminaries which is used to do the comparative statics. The second subsection looks at the comparative statics with a higher interest rate. The last subsection examines the consequences of an increase in the tax-rate.

5.1 Preliminaries

The parameters of interest are the wage, the employment and the welfare of the union members. Taking the differential of the parameters of interest yields

$$dw = (db + d\alpha)(1+\beta) + (b+\alpha)\, d\beta \tag{27}$$

$$dL = -\frac{1}{2(\alpha+b)^3}(d\alpha + db) \tag{28}$$

$$dU = db\,(1-\beta) + d\beta\,(J-b) + \beta dJ \tag{29}$$

The endogenous variables b, β and J has the following differentials

$$d\beta = \frac{(dr\,(1-\tau) - r \cdot d\tau)\, r_U - dr_U \cdot r\,(1-\tau)}{\left[r\,(1-\tau) + r_U\right]^2} \tag{30}$$

$$db = \frac{1}{r_U}\left(du_b - b \cdot dr_U\right) \tag{31}$$

$$dJ = db\,(1-L) - Ld\alpha \tag{32}$$

From (32) it is clear that the three effects affecting the Nash bargaining solution are in fact only two. There is no way the line through b can be shifted without the Pareto frontier shifting as well. An increase in the inside option for the union shifts the Pareto frontier outward.

5.2 Increase in interest rate

If the interest rate increases, then the wages increases by

$$\frac{dw}{dr} = (b + \alpha)\frac{d\beta}{dr} \tag{33}$$

which is strictly positive, because $\frac{d\beta}{dr} = \frac{(1-\tau)r_U}{[r(1-\tau)+r_U]^2} > 0$, which follows from (30). The higher interest rate makes it more expensive for the firm to not reject an offer. If the firm rejects an offer at time t, then it needs to wait Δ time before making a counteroffer. The interest rate is the gain the firm could have made on its share by accepting the offer at time t, so the higher eagerness for the firm leads to the union having a stronger bargaining position.

The employment level is independent of the interest rate, because in the efficient bargaining model, the firm employs according to (7). The unemployment level therefore isn't affected by changes to the interest rate.

The change in welfare for an increase in r is

$$\frac{dU}{dr} = \frac{d\beta}{dr}(J - b) \tag{34}$$

which is increasing because of the increased bargaining power. This represent a left rotation of the line through b in figure 1.

5.3 Increase in taxes

When the taxes increases, the wage changes according to

$$\frac{dw}{d\tau} = \frac{db}{d\tau}(1 + \beta) + (b + \alpha)\frac{d\beta}{d\tau} \tag{35}$$

which is strictly positive (**Proof: See appendix B**).

If the taxes increases, then the employment decreases according to

$$\frac{dL}{d\tau} = \frac{dL}{db}\frac{db}{d\tau} \tag{36}$$

which is strictly negative.

The welfare changes by a higher tax rate by

$$\frac{dU}{d\tau} = \frac{db}{d\tau}(1-\beta) + \frac{d\beta}{d\tau}(J-b) + \beta\frac{dJ}{d\tau} \tag{37}$$

which is positive because the outward shift of the line through b and the Pareto frontier over compensates for the fall in utility by the right rotation of the line through b (**See appendix C for calculations**).

6 Falling membership

In this section the effect of a falling membership will be analyzed in detail. The first subsection will introduce the empirical findings on falling union membership. The second subsection will analyze the consequences of a falling membership. The third subsection will examine the consequences of a falling membership if the government adjusts taxes to keep unemployment constant.

6.1 Empirical findings of a falling membership

The evolution of members in the Danish trade union's have been falling for some time. In figure 2 the evolution of the danish union members is decreasing with time, with a huge drop from 2011 to 2012, and an average drop in members from 2.05 million in 2007 to 1.86 million members in 2019.

Figure 2: Membership evolution in Denmark

Source: Statistikbanken, table: LONMED2.
Downloaded April 28, 2021

6.2 Consequences without government involvement

Taking the derivative of M with respect to μ yields

$$\frac{dM}{d\mu} = -\frac{\alpha}{(1-\mu)^2} < 0 \tag{38}$$

which has the following interpretation. If μ increases, then the workers disutility from free-riding decreases, and so for a constant level of α, more workers will free-ride. The fall in the membership hurts the union through its cost of time. The derivative of r_U with respect to μ yields

$$\frac{dr_U}{d\mu} = \left(\frac{\alpha}{1-\mu}\right)^2 > 0 \tag{39}$$

The increase in the cost of time is dependent on the membership, and will for a low membership increase more than for a high membership. This is clear from inserting the expression for $M = 1 - \frac{\alpha}{1-\mu}$ into (39), which yields

$$\frac{dr_U}{d\mu} = (1-M)^2$$

For a constant interest rate the increase in μ, meaning a fall in reputational costs, yields

$$\frac{dw}{d\mu} = \frac{db}{d\mu}(1+\beta) + (b+\alpha)\frac{d\beta}{d\mu} \tag{40}$$

which is strictly negative. There is two effects that decreases the wages for lower reputational costs. The first term in (40) is the fall in wages for a higher cost of time, r_U, because it decreases the inside option, so the union discount future payoffs higher, which means the gain from disagreeing perpetually falls. The second term is the fall in wages because the bargaining power decreases.

The employment changes because the inside option depends on μ. Taking the derivative of labor with respect to μ yields

$$\frac{dL}{d\mu} = \frac{dL}{db} \cdot \frac{db}{d\mu} \tag{41}$$

which is positive since $\frac{db}{d\mu} < 0$ from (31) and $\frac{dL}{db} < 0$ from (28).
The unemployment falls with a lower reputational cost, because the fewer members decreases the total union dues collected, so the union's inside option decreases. The lower inside option increases employment.

The Pareto frontier decreases by $\frac{dJ}{d\mu} = \frac{db}{d\mu}(1-L)$ from (32). This means that the welfare decreases by

$$\frac{dU}{d\mu} = \frac{db}{d\mu}(1-\beta) + \frac{d\beta}{d\mu}(J-b) + \beta\frac{dJ}{d\mu} \tag{42}$$

$$= \frac{db}{d\mu}(1-\beta L) + \frac{d\beta}{d\mu}(J-b) \tag{43}$$

6.3 Consequences of government involvement

To keep the unemployment constant at some level, u^n, the government adjusts the tax rate such that $1 - L = u^n$, which imply

$$\tau^* = \frac{r_U}{r}\left(\underbrace{\frac{1}{2\sqrt{1-u^n}}}_{\equiv b^n} - \alpha\right) \tag{44}$$

(**See appendix D for calculations**)

When the reputational costs decreases, then the tax rate changes by

$$\frac{d\tau}{d\mu} = \frac{(1-M)^2}{r} b^n \tag{45}$$

which is strictly positive. The increase in taxes decreases the bargaining power, but raises the inside option. If the membership is falling, then the unemployment falls below u^n, and so the taxes increases to compensate for the fall.

If the membership falls, then to keep the unemployment level at u^n the government increases the tax rate. The effect on the wages is

$$\frac{dw|_{\tau=\tau^*}}{d\mu} = (b^n + \alpha)\frac{d\beta|_{\tau=\tau^*}}{d\mu} \tag{46}$$

which is strictly negative because

$$\frac{d\beta|_{\tau=\tau^*}}{d\mu} = -\frac{r \cdot \frac{d\tau}{d\mu} r_U + \frac{dr_U}{d\mu} \cdot r(1-\tau)}{\left[r(1-\tau) + r_U\right]^2} \tag{47}$$

$$= -(1-M)^2 \frac{b^n r_U + r(1-\tau)}{\left[r(1-\tau) + r_U\right]^2} \tag{48}$$

The whole fall in wages is through the bargaining power. The bargaining power falls more than under government intervention, because the taxes increases to keep unemployment at u^n. The fall in wages with government intervention is smaller than without intervention (**Proof: See appendix E**).

The inside option at the optimal tax rate, $b|_{\tau=\tau^*}$, doesn't depend on μ. The surplus depends on the inside option, the union dues and the employment level. They are all constant with an optimal tax rate. Taking the derivative of U at the optimal tax rate then yields

$$\frac{dU|_{\tau=\tau^*}}{d\mu} = \frac{db|_{\tau=\tau^*}}{d\mu} + \frac{d\beta|_{\tau=\tau^*}}{d\mu}(J-b) + \beta\left(\frac{dJ|_{\tau=\tau^*}}{d\mu} - \frac{db|_{\tau=\tau^*}}{d\mu}\right) \tag{49}$$

$$= \frac{d\beta|_{\tau=\tau^*}}{d\mu}(J-b) \tag{50}$$

which means that the whole fall in utility for lower reputational costs is caused by the lower bargaining power. This is because the employment depends on b, and so the government raises the unemployment benefit, u_b, proportional to the fall in r_U.

The fall in bargaining power when the government keeps unemployment at u^n is higher than under no government involvement. The governmental rule benefits the workers when

$$\frac{dU}{d\mu} < \frac{dU|_{\tau=\tau^*}}{d\mu} \iff (1-\beta L) > \frac{(1-\beta)^2}{4(\alpha+b)} \tag{51}$$

(**See appendix F for calculations**)

The inequality always hold because of (8). The fall in welfare when membership falls is greater without government involvement than with.

7 Conclusion

The purpose of this thesis was to examine a union-firm bargaining model with endogenized bargaining power. The bargaining power depends on the after tax interest rate and the membership of the union. A higher tax decreases the bargaining power for the union, because the firm becomes less impatient. A higher membership increases the bargaining power for the union, because the increase in union dues enables the union to strike for longer. The membership is decided by the reputational costs of not joining the union, where the marginal member represent the size of the union. The tax rate is decided by the government. A decrease in reputational costs decreases the membership for the union. The wages, unemployment and welfare for the union members decreases as a consequence. If the government's objective is to keep unemployment constant, then a fall in membership still decreases wages and welfare, but by a lesser degree than if the government didn't intervene. A prediction of this model is that the union wage-differential is decreasing in the interest rate, but increasing in taxes on interest rates. Another prediction is that pressure on non-members to join the union increases the union membership. A further exploration of the setup in a dynamic framework is of macroeconomic interest, because of the prediction that the after-tax interest rate increases the wage. If this holds in a dynamic setup, then it predicts that for a low interest rate, the wage-growth is low and so the inflation will also be low. A consequence might be that an increase in the money supply when the interest rate is low doesn't increase inflation significantly.

Appendix

A

Inserting the utility function for the union in the Nash bargaining solution yields

$$\begin{aligned} L\left(w-\alpha\right)+\left(1-L\right)b &= b+\beta\left(\sqrt{L}-L\alpha+\left(1-L\right)b+b\right) \\ L\left(w-\alpha-b\right) &= \beta\left(\sqrt{L}-L\left(\alpha+b\right)\right) \\ w-\alpha-b &= \beta\left(\frac{1}{\sqrt{L}}-\left(\alpha+b\right)\right) \\ w &= \left(\alpha+b\right)\left(1-\beta\right)+\beta\frac{1}{\sqrt{\frac{1}{4\left(\alpha+b\right)^2}}} \\ &= \left(\alpha+b\right)\left(1-\beta\right)+2\beta\left(\alpha+b\right) \\ &= \left(\alpha+b\right)\left(1+\beta\right) \end{aligned}$$

B

I take the derivative of w with respect to τ, which by some algebraic manipulations yields.

$$\begin{aligned}
\frac{dw}{d\tau} &= \frac{db}{d\tau}(1+\beta) + (b+\alpha)\frac{d\beta}{d\tau} \\
&= \frac{r}{r_U}(1+\beta) - \frac{r r_U}{\left[r(1-\tau)+r_U\right]^2}\left(\frac{\tau r}{r_U}+\alpha\right) > 0 \\
1+\beta &> \frac{r_U^2}{\left[r(1-\tau)+r_U\right]^2}\left(\frac{\tau r}{r_U}+\alpha\right) \\
1+\beta &> (1-\beta)\left(\frac{\tau r}{r(1-\tau)+r_U} + \frac{r_U}{r(1-\tau)+r_U}\alpha\right) \\
1+\beta &> (1-\beta)\left(\frac{\tau r}{r(1-\tau)+r_U} + (1-\beta)\alpha\right) \\
1+\beta &> (1-\beta)\left(\frac{\tau r}{r(1-\tau)+r_U} + (1-\beta)\alpha\right)
\end{aligned}$$

The term $\frac{\tau r}{r(1-\tau)+r_U}$ is increasing in τ. At the highest tax rate it is equal to $\frac{r}{r+r_U} = \beta$. This implies

$$1+\beta > (1-\beta)(\beta+(1-\beta)\alpha)$$

which always hold since $\beta + (1-\beta)\alpha < 1$.

C

Taking the derivative of U with respect to τ yields

$$\frac{dU}{d\tau} = \frac{db}{d\tau}(1-\beta) + \frac{d\beta}{d\tau}(J-b) + \beta\frac{dJ}{d\tau}$$

Taking the derivative of J with respect to τ yields

$$\frac{dJ}{d\tau} = \frac{db}{d\tau}(1-L)$$

Inserting this into the expression for the $\frac{dU}{d\tau}$ yields

$$
\begin{aligned}
\frac{dU}{d\tau} &= \frac{db}{d\tau}\left(1-\beta L\right) + \frac{d\beta}{d\tau}\left(J-b\right) > 0 \\
\Rightarrow \frac{r}{r_U}\left(1-\beta L\right) &> \frac{r r_U}{\left[r\left(1-\tau\right)+r_U\right]^2}\left(J-b\right) \\
\Rightarrow \left(1-\beta L\right) &> \left(1-\beta\right)\left(J-b\right) \\
\Rightarrow 4\left(\alpha+b\right) &> \frac{\left(1-\beta\right)}{\left(1-\beta L\right)}
\end{aligned}
$$

which always holds since $\alpha + b \geq \frac{1}{2}$ and $\frac{(1-\beta)}{(1-\beta L)} < 1$.

D

Setting the unemployment, $1-L$, equal to the unemployment goal, u^n, yields

$$
\begin{aligned}
1 - L &= u^n \\
\frac{1}{4\left(\alpha+b\right)^2} &= 1 - u^n \\
b &= \frac{1}{2\sqrt{1-u^n}} - \alpha \\
\tau^* &= \frac{r_U}{r}\left(\frac{1}{2\sqrt{1-u^n}} - \alpha\right)
\end{aligned}
$$

E

Setting the derivatives of wages with respect to μ for the case with and without government intervention against each other yields

$$\begin{aligned}
\frac{dw|_{\tau=\tau^*}}{d\mu} &> \frac{dw}{d\mu} \\
\left(b^n+\alpha\right)\frac{d\beta|_{\tau=\tau^*}}{d\mu} &> \frac{db}{d\mu}\left(1+\beta\right)+\left(b+\alpha\right)\frac{d\beta}{d\mu} \\
-\left(b^n+\alpha\right)\left(1-M\right)^2\frac{b^n r_U + r\left(1-\tau\right)}{\left[r\left(1-\tau\right)+r_U\right]^2} &> -b\frac{1}{r_U}\left(1-M\right)^2\left(1+\beta\right)-\left(b+\alpha\right)\left(1-M\right)^2\frac{r\left(1-\tau\right)}{\left[r\left(1-\tau\right)+r_U\right]^2} \\
\left(b^n+\alpha\right)\frac{b^n r_U + r\left(1-\tau\right)}{\left[r\left(1-\tau\right)+r_U\right]^2} &< b\frac{1}{r_U}\left(1+\beta\right)+\left(b+\alpha\right)\frac{r\left(1-\tau\right)}{\left[r\left(1-\tau\right)+r_U\right]^2} \\
\left(b^n+\alpha\right)\frac{b^n r_U}{\left[r\left(1-\tau\right)+r_U\right]^2} &< b\frac{1}{r_U}\left(1+\beta\right) \\
\left(b^n+\alpha\right)\left(1-\beta\right)^2 &< 1+\beta \\
\left(\frac{\tau^* r}{r_U}+\alpha\right)\left(1-\beta\right)^2 &< 1+\beta \\
\left(1-\beta\right)^2 &< 1+\beta
\end{aligned}$$

which always holds because $\beta \in [0,1]$.

F

Inserting the expression for the derivatives of U with respect to μ in the case with and without government involvement yields

$$\begin{aligned}
\frac{dU}{d\mu} &< \frac{dU|_{\tau=\tau^*}}{d\mu} \\
\frac{db}{d\mu}\left(1-\beta L\right)-\frac{dr_U}{d\mu}\frac{r\left(1-\tau\right)}{\left[r\left(1-\tau\right)+r_U\right]^2}\left(J-b\right) &< -r\frac{r_U\frac{d\tau}{d\mu}+\frac{dr_U}{d\mu}\left(1-\tau\right)}{\left[r\left(1-\tau\right)+r_U\right]^2}\left(J-b\right) \\
\frac{db}{d\mu}\left(1-\beta L\right) &< -r\frac{r_U\frac{d\tau}{d\mu}}{\left[r\left(1-\tau\right)+r_U\right]^2}\left(J-b\right) \\
-b\frac{(1-M)^2}{r_U}\left(1-\beta L\right) &< -\frac{r_U(1-M)^2 b}{\left[r\left(1-\tau\right)+r_U\right]^2}\left(J-b\right) \\
\left(1-\beta L\right) &> \frac{\left(1-\beta\right)^2}{4\left(\alpha+b\right)}
\end{aligned}$$

References

Black, D. (1948). On the rationale of group decision-making. *Journal of political economy*, *56*(1), 23–34.

Booth, A. L. (1995). *The economics of the trade union*. Cambridge University Press.

Booth, A. L., & Chatterji, M. (1993). Reputation, membership and wages in an open shop trade union. *Oxford Economic Papers*, 23–41.

Manning, A. (1987). An integration of trade union models in a sequential bargaining framework. *The Economic Journal*, *97*(385), 121–139.

Muthoo, A. (1999). *Bargaining theory with applications*. Cambridge University Press.